Math 10460 - Honors Mathematics II
Homework 12b - Due Wednesday, April 20

You must show your work in all of the problems!!!

Recall that S^2 is the sphere, $\mathbb{T}$ is the torus, and $\mathbb{K}$ is the Klein bottle.

(2) Using the fact that
$$\chi(M\#N) = \chi(M) + \chi(N) - 2,$$
show that
  (a) $\chi(g\mathbb{T}) = 2 - 2g$
  (b) $\chi(m\mathbb{P}) = 2 - m$
  (Remember, $\chi(T) = 2$ and $\chi(\mathbb{P}) = 1$.)

(3) Let Σ be a surface and suppose $\chi(\Sigma) = -3$. Use the classification theorem to identify Σ as S^2, $g\mathbb{T}$ for some g, or $m\mathbb{P}$ for some m. (You must specify the value of g or m in either of those cases, and explain how you obtained it.)

(4) Suppose the surface Σ can be represented as a polygon with word given by
$$ab^{-1}a^{-1}cb^{-1}c$$
Is Σ orientable? Why?

(5) Show that $m\mathbb{P}$ can be written as either $k\mathbb{T}\#\mathbb{P}$ or $k\mathbb{T}\#\mathbb{K}$ for some integer k. What do you notice about the Euler characteristic in the two cases?

(6) In relation to the previous problem, why do you think we write $m\mathbb{P}$ in the classification theorem for the non-orientable surfaces rather than split it into the two cases $k\mathbb{T}\#\mathbb{P}$ and $k\mathbb{T}\#\mathbb{K}$.